

NAG Toolbox for MATLAB

f08fa

1 Purpose

f08fa computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric matrix A .

2 Syntax

```
[a, w, info] = f08fa(jobz, uplo, a, 'n', n)
```

3 Description

The symmetric matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – string

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The n by n symmetric matrix A .

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix *A*.

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

If **jobz** = 'V', then if **info** = 0, **a** contains the orthonormal eigenvectors of the matrix *A*.

If **jobz** = 'N', then on exit the lower triangle (if **uplo** = 'L') or the upper triangle (if **uplo** = 'U') of **a**, including the diagonal, is destroyed.

2: **w(*)** – double array

Note: the dimension of the array **w** must be at least $\max(1, n)$.

If **info** = 0, the eigenvalues in ascending order.

3: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = *i*, the algorithm failed to converge; *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The complex analogue of this function is f08fn.

9 Example

```
jobz = 'Vectors';
uplo = 'Upper';
a = [1, 2, 3, 4;
     0, 2, 3, 4;
     0, 0, 3, 4;
     0, 0, 0, 4];
[aOut, w, info] = f08fa(jobz, uplo, a)

aOut =
    0.7003   -0.5144    0.2767    0.4103
    0.3592    0.4851   -0.6634    0.4422
   -0.1569    0.5420    0.6504    0.5085
   -0.5965   -0.4543   -0.2457    0.6144
w =
   -2.0531
   -0.5146
   -0.2943
   12.8621
info =
      0
```